Name: ______________________________

MATH 107.01
EXAM II

This is a closed book closed notes exam. The use of calculators is not permitted. Show all work and clearly indicate your final answers. If all your work is not shown, you will not receive full credit. The last two pages are for scratch work. If you run out of room for a problem feel free to use the scratch paper, but be sure to be clear about where the work is for each problem. The point value of each question is indicated in parentheses. Make sure that you sign the Duke Honor Code at the bottom of this page. Good luck!

Problem	**Points**	**Score**
1	8	
2	12	
3	12	
4	14	
5	18	
6	10	
7	8	
8	6	
9	12	
Total	100	

Honor Code: I have completed this exam in the spirit of the Duke Community Standard and have neither given nor received aid.

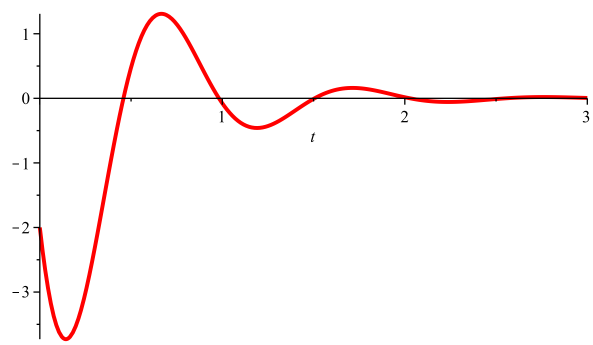
Signature: ______________________________

Problem 1 (8 Points)**.** An object of mass 1 is hung on a spring of spring constant 40 and is suspended in oil with friction coefficient 4. No external force is applied and the object is given an initial velocity of 25 from its equilibruim position.

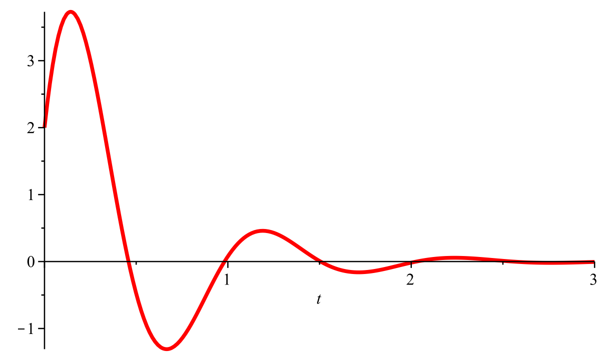
(a) Circle the differential equation satisfied by the position $s(t)$ of the object.

$40s'' + 4s' + s = 0$	$4s'' + 40s' + s = 0$
$4s'' + s' + 40s = 0$	$s'' + 4s' + 40s = 0$
$40s'' + s' + 4s = 0$	$s'' + 40s' + 4s = 0$

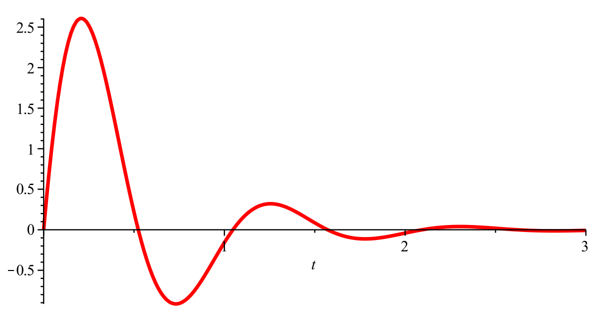
(b) Circle the graph depicting the motion of the object.



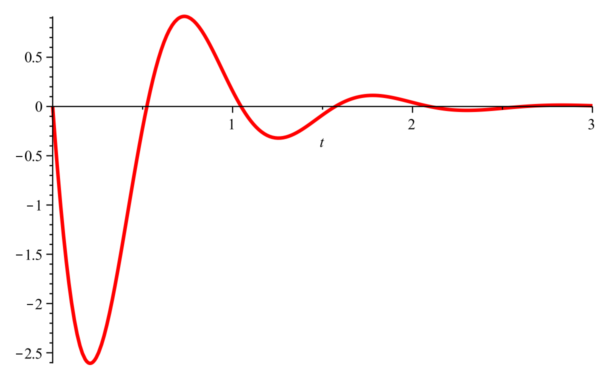
(a)



(b)



(c)



(d)

Problem 2 (12 Points)**.** Solve the initial value problem

$$\begin{aligned} y''' - y' &= 0 \\ y(0) &= 1 \\ y'(0) &= 1 \\ y''(0) &= 0. \end{aligned}$$

Problem 3 (12 Points)**.** Find the general solution to the system

$$Y'(t) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} Y(t).$$

Problem 4 (14 Points)**.** Let $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ where $b \neq 0$.

(a) Show that the eigenvalues of A are $\lambda_1 = a + b$ and $\lambda_2 = a - b$.

(b) Find bases for the eigenspaces of A.

(c) Is A diagonalizable? If so, find a matrix P and a diagonal matrix Λ such that $\Lambda = P^{-1}AP$.

Problem 5 (18 Points)**.** Let α and β be the bases for P_1 given by

$$\alpha = \{x, 1\}$$
$$\beta = \{x+1, x-1\}$$

and let $\mathbf{1}, T : P_1 \to P_1$ be the linear transformations defined by

$$\mathbf{1}(ax+b) = ax+b$$
$$T(ax+b) = (a+2b)\,x + (2a+b)\,.$$

(a) Show that $\mathbf{1}T = T\mathbf{1} = T$.

(b) Find $[\mathbf{1}]_\beta^\alpha$ and $[\mathbf{1}]_\alpha^\beta$.

(c) Find $[T]_\alpha^\alpha$.

(d) Express each of $[T]_\beta^\beta$, $[T]_\alpha^\beta$, and $[T]_\beta^\alpha$ in terms of $[\mathbf{1}]_\alpha^\beta$, $[\mathbf{1}]_\beta^\alpha$, and $[T]_\alpha^\alpha$. Do not substitute in the matrices you computed in (b) and (c), just do this symbolically.

(d) Is T diagonalizable? Why or why not?

Problem 6 (10 Points)**.** Let $A, B \in M_{n\times n}(\mathbb{R})$ be similar. Show that A and B have the same characteristic polynomial.

Problem 7 (8 Points)**.** Write the differential equation

$$y^{(3)} + 2xy'' + e^x y = 5$$

as a system of first order differential equations in matrix form.

Problem 8 (6 Points)**.** A linear transformation $T : V \to W$ is called **injective** if $T(v) = T(w)$ implies $v = w$.

(a) Suppose that T is injective. Show that $\ker(T)$ consists only of the zero vector.

Hint: Start by letting v be an arbitrary vector in $\ker(T)$ and then show that $v = \mathbf{0}$.

(b) Suppose that $\ker(T)$ consists only of the zero vector. Show that T is injective.

Problem 9 (12 Points). Find a particular solution to the nonhomogeneous system

$$Y'(t) = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} Y(t) + \begin{bmatrix} 4 \\ 6 \end{bmatrix}.$$

Name: ______________________________

MATH 107.01 Exam I Scratch Work

Name: ______________________

MATH 107.01 Exam I Scratch Work