1. Find the Taylor polynomial, centered at \( x = a \), of degree \( n \) for each of the following functions (you can use these derivations for the homework from section 8.8):

   (a) \( f(x) = e^{x^2} \), \( a = 0 \), \( n = 3 \)
   (b) \( f(x) = x \sin(x) \), \( a = 0 \), \( n = 4 \)
   (c) \( f(x) = x \ln(x) \), \( a = 1 \), \( n = 3 \)
   (d) \( f(x) = \sqrt{1 + x} \), \( a = 1 \), \( n = 3 \)
   (e) \( f(x) = \tan(x) \), \( a = \frac{\pi}{4} \), \( n = 2 \)

2. Give an example of a function \( f(x) \), such that the Taylor polynomial of degree 4 of \( f \) is the same as the Taylor polynomial of degree \( n \) for all \( n > 4 \).

3. Consider the function \( f(x) \) graphed below:

   ![Graph of f(x)](image)

Determine whether each of the following could be the 2nd degree Taylor polynomial of \( f(x) \) centered at \( x = 1 \). Explain your answers.

   (a) \( (x - 1) - (x - 1)^2 \)
   (b) \( 1 - 3(x - 1) + \frac{(x - 1)^2}{4} \)
   (c) \( -1 + \frac{(x - 1)}{2} - 4(x - 1)^2 \)
   (d) \( 1 + (x - 1) - \frac{(x - 1)^2}{3} \)
   (e) \( 1 - 2(x - 1)^2 \)

4. The table below gives information about a continuous function \( f(x) \):

   \[
   \begin{array}{c|c|c|c|c}
   f(0) & f'(0) & f''(0) & f'''(0) & f^{(4)}(0) \\
   \hline
   0 & 1 & -3 & 7 & -15
   \end{array}
   \]
(a) Use a 4th degree Taylor polynomial to estimate $f(0.1)$.

(b) Use a 4th degree Taylor polynomial to estimate $\int_0^{0.5} f(x) \, dx$.

5. Suppose $f'(x) = \frac{(f(x))^2}{x}$, $f(1) = 2$

(a) Find the 2nd degree Taylor polynomial for $f(x)$, centered at $x = 1$.

(b) Use your answer to (a) to find $\lim_{x \to 1} \frac{f(x) - 2}{x - 1}$. 