Homework - Taylor Polynomials

1. Find the Taylor polynomial, centered at $x = a$, of degree n for each of the following functions (you can use these derivations for the homework from section 8.8):

   (a) $f(x) = e^{x^2}$, $a = 0$, $n = 3$

   (b) $f(x) = x\sin(x)$, $a = 0$, $n = 4$

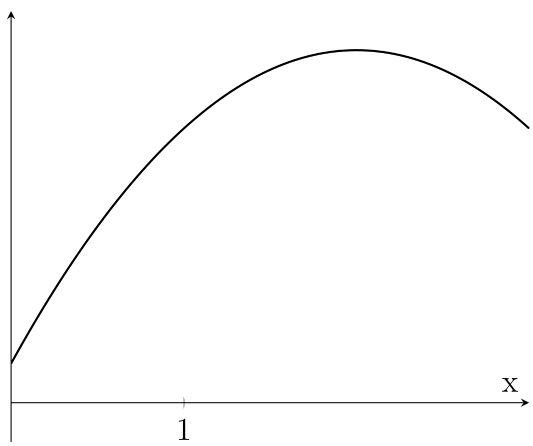
   (c) $f(x) = x\ln(x)$, $a = 1$, $n = 3$

   (d) $f(x) = \sqrt{1+x}$, $a = 1$, $n = 3$

   (e) $f(x) = \tan(x)$, $a = \frac{\pi}{4}$, $n = 2$

2. Give an example of a function $f(x)$, such that the Taylor polynomial of degree 4 of f is the same as the Taylor polynomial of degree n for all $n > 4$.

3. Consider the function $f(x)$ graphed below:



   Determine whether each of the following could be the 2nd degree Taylor polynomial of $f(x)$ centered at $x = 1$. Explain your answers.

   (a) $(x-1) - (x-1)^2$

   (b) $1 - 3(x-1) + \frac{(x-1)^2}{4}$

   (c) $-1 + \frac{(x-1)}{2} - 4(x-1)^2$

   (d) $1 + (x-1) - \frac{(x-1)^2}{3}$

   (e) $1 - 2(x-1)^2$

4. The table below gives information about a continuous function $f(x)$:

$f(0)$	$f'(0)$	$f''(0)$	$f'''(0)$	$f^{(4)}(0)$
0	1	-3	7	-15

(a) Use a 4th degree Taylor polynomial to estimate $f(0.1)$.

(b) Use a 4th degree Taylor polynomial to estimate $\int_0^{0.5} f(x)\,dx$.

5. Suppose $f'(x) = \dfrac{\left(f(x)\right)^2}{x}, \qquad f(1) = 2$

(a) Find the 2nd degree Taylor polynomial for $f(x)$, centered at $x = 1$.

(b) Use your answer to (a) to find $\lim_{x \to 1} \dfrac{f(x) - 2}{x - 1}$.