Definition 1. A graph is a tuple $\gamma = (V(\gamma), H(\gamma), \pi, \sigma)$ consisting of
- a finite set $V(\gamma)$ whose elements are called the vertices of $\gamma$
- a finite set $H(\gamma)$ whose elements are called the half edges of $\gamma$
- a map $\pi: H(\gamma) \rightarrow V(\gamma)$
- an involution $\sigma$ on $H(\gamma)$.

Definition 2. Let $\gamma = (V(\gamma), H(\gamma), \pi, \sigma)$ be a graph.
- The edges of $\gamma$ are the elements of $E(\gamma) = \text{orb}(\sigma)$.
- The tails of $\gamma$ are the elements of $T(\gamma) = \text{fix}(\sigma)$.
- The connected components of $\gamma$ are the elements of $C(\gamma) = V(\gamma)/\sim$ where $\sim$ is the smallest equivalence relation satisfying $v \sim w$ if there exists an $h \in H(\gamma)$ such that $\pi(h) = v$ and $\pi(\sigma(h)) = w$.
- The tail map of $\gamma$ is the map $\iota: T(\gamma) \rightarrow C(\gamma)$ given by $\iota(h) = [\pi(h)]$.
- The path relation on $\gamma$ is the relation $\sim_p$ on $V(\gamma)$ given by $v \sim_p w$ if and only if there exists a map $f: [2n] \rightarrow H(\gamma)$ such that
  - $\{\{f(k), f(k+1)\} : 0 \leq k \leq 2n - 2\} \subseteq E(\gamma)$
  - $f(0) = v$
  - $f(2n - 1) = w$.

Definition 3. A forest is a graph $\gamma = (V(\gamma), H(\gamma), \pi, \sigma)$ such that the fibres of $\pi$ are of cardinality at least three and whose path relation is irreflexive.

Definition 4. A rooted forest is a pair $(\gamma, s)$ consisting of a forest $\gamma$ and a section $s$ of the tail map of $\gamma$.

Definition 5. Let $\text{Graphs}$ be the category where
- $\text{ob}(\text{Graphs}) = \text{ar}((\text{set} \downarrow \text{set})/\approx$
- $\text{ar}(\text{Graphs})$ is the collection of all graphs
- $\text{dom}(\gamma) = [\pi_\gamma]$
- $\text{cod}(\gamma) = [\iota_\gamma]$
- $1_f$ is a graph $(V, H, \pi, \sigma)$ satisfying
  - $V \approx \text{cod}(f)$
  - $H \approx \text{dom}(f)$